

DOCUMENT RESUME

ED 480 463

SE 068 319

AUTHOR Leigh-Lancaster, David
TITLE The Victorian Curriculum and Assessment Authority
Mathematical Methods Computer Algebra Pilot Study and
Examinations.
PUB DATE 2003-06-23
NOTE 34p.; Produced by the Victorian Curriculum and Assessment
Authority. Paper presented at the CAME Conference (3rd,
Rheims, France, June 23-24, 2003).
PUB TYPE Reports - Research (143)
EDRS PRICE EDRS Price MF01/PC02 Plus Postage.
DESCRIPTORS *Academic Achievement; *Computer Assisted Instruction;
Computer Uses in Education; Evaluation; Mathematical
Applications; Mathematical Concepts; Mathematics Curriculum;
*Mathematics Instruction; Secondary Education
IDENTIFIERS Computer Algebra

ABSTRACT

Computer algebra systems (CAS) have now become much more readily accessible for use in secondary school mathematics on both hand-held and computer platforms. While the initial focus of work with CAS from the early 1980's has generally been with respect to pedagogical and curriculum issues, as familiarity with CAS in senior secondary mathematics contexts has evolved around the world, systems and organizations have responded in various ways to the increasing availability of CAS and its impact on assessment, in particular end of secondary schooling formal examinations. This paper discusses key design and development aspects of the first examinations for the VCAA Mathematical Methods (CAS) pilot study in 2002, and provides some preliminary analysis and commentary with respect to student performance on these examinations. (Author)

Reproductions supplied by EDRS are the best that can be made
from the original document.

**The Victorian Curriculum and Assessment Authority
Mathematical Methods Computer Algebra Pilot Study and
Examinations**

David Leigh-Lancaster

U.S. DEPARTMENT OF EDUCATION
Office of Educational Research and Improvement
EDUCATIONAL RESOURCES INFORMATION
CENTER (ERIC)

This document has been reproduced as
received from the person or organization
originating it.

Minor changes have been made to improve
reproduction quality

Points of view or opinions stated in this
document do not necessarily represent
official OERI position or policy.

PERMISSION TO REPRODUCE AND
DISSEMINATE THIS MATERIAL HAS
BEEN GRANTED BY

D. Leigh-Lancaster

TO THE EDUCATIONAL RESOURCES
INFORMATION CENTER (ERIC)

1

The Victorian Curriculum and Assessment Authority Mathematical Methods computer algebra pilot study and examinations

David Leigh- Lancaster

Victorian Curriculum and Assessment Authority

Abstract

Computer algebra systems (CAS) have now become much more readily accessible for use in secondary school mathematics on both hand-held and computer platforms. While the initial focus of work with CAS from the early 1980's has generally been with respect to pedagogical and curriculum issues, as familiarity with CAS in senior secondary mathematics contexts has evolved around the world, systems and organisations have responded in various ways to the increasing availability of CAS and its impact on assessment, in particular end of secondary schooling formal examinations. This paper discusses key design and development aspects of the first examinations for the VCAA Mathematical Methods (CAS) pilot study in 2002, and provides some preliminary analysis and commentary with respect to student performance on these examinations.

Background

While earlier considerations on the use of CAS in mathematics education focussed on pedagogical and curriculum issues, these issues do not arise in isolation from assessment (see Leigh-Lancaster and Stephens, 1997, 2001). The notion of *congruence* between pedagogy, curriculum and assessment is a central part of the discourse on the use of such technology (see Leigh-Lancaster, 2000, HREF1). Here congruence refers to the alignment between curriculum structure and aims, approaches to working mathematically, and the nature and purpose of assessments, in particular examinations. The use of technology in the senior mathematics curriculum, and end of secondary schooling mathematics examinations in Victoria, has evolved over the last several decades as different technologies have become more widely available and integrated into mainstream teaching and learning practice:

- 1970 – slide rule and four figure mathematical tables;
- 1978 – scientific calculators;
- 1997/8 – approved graphics calculators permitted (examinations graphics calculator 'neutral');
- 1999 – 'assumed access' for graphics calculators in Mathematical Methods and Specialist Mathematics examinations, permitted for Further Mathematics examinations;

- 2000 – ‘assumed access’ for graphics calculators in all mathematics examinations, examinations for revised Victorian Certificate of Education (VCE) Mathematics study 2000 – 5 incorporating some graphics calculator ‘active’ questions;
- November 2002 – Mathematical Methods (CAS) pilot study, ‘assumed access’ for approved CAS in pilot examinations.

Government policies, directions and resources for various sectors related to ICT, reflect cognisance of the role that such technologies play in the economy, education and society in general (see, for example, *it reality bytes*, 2001, HREF2, and the *Ministerial Statement on Knowledge and Skills for the Innovation Society*, HREF3). Particular curriculum and assessment projects, such as the VCAA Mathematical Methods (CAS) pilot, necessarily occur within the context of the corresponding policy framework, for example, the VCAA is explicit in its *Strategic Plan 2002 – 2004* that it will ensure ICT and innovative thinking are embedded throughout the curriculum (HREF4). Important ICT related issues in this context are the potential for flexible delivery of curriculum and assessment in senior certificates, support for student engagement, and social justice considerations of equity, access and retention.

Mathematical Methods (CAS) Units 1 - 4 is an accredited pilot study of the Victorian Curriculum and Assessment Authority for the period from January 2001 - December 2005. The pilot study is monitored and evaluated as part of the ongoing review and accreditation of VCE studies, and, following on from a successful conclusion to the pilot, there would likely be a subsequent period of overlapping accreditation for the revised Mathematical Methods and Mathematical Methods (CAS) courses. Details of the pilot, including the study design for Units 1 – 4, sample and 2002 examinations, assessment reports and other teacher resources, can be accessed from the VCAA website (HREF5).

The first phase of the VCAA pilot study 2000 – 2002, involved students from three volunteer Stage 1 schools, and was implemented in conjunction with the CAS - CAT project, a research partnership between the VCAA, the Department of Science and Mathematics Education at the University of Melbourne, and three calculator companies (CASIO, Hewlett-Packard, and Texas Instruments). The CAS - CAT project has been funded by a major Commonwealth Australian Research Council (ARC) Strategic Partnership with Industry Research and Training (SPIRT) grant (HREF6). In November 2002, 78 students from the three Stage 1 schools sat end of year final Mathematical Methods (CAS) Unit 3 and 4 examinations, for which student access to an approved CAS calculator (TI-89, CASIO ALGEBRA FX 2.0 or HP 40G) was assumed.

The second and third stages of the VCAA expanded pilot study 2001 – 2005, incorporate the original three schools (implementing Mathematical Methods (CAS) Units 1 and 2 from 2001 and Units 3 and 4 from 2002) and include two additional groups: nine volunteer Stage 2 schools implementing Units 1 and 2 from 2002 and Units 3 and 4 from 2003, and a further seven volunteer Stage 3 schools implementing Units 1 and 2 from 2002 and Units 3 and 4 from 2004. The schools in the expanded pilot include co-educational and single sex, metropolitan and regional schools from government, catholic and independent sectors, using a range of different CAS. Thus, there will be slightly over 250 students enrolled in Units 3 and 4 from 11 schools of the expanded pilot in 2003. This will include students using the CAS TI Voyage 200, Derive and *Mathematica* in one school for each of these CAS.

Early use of CAS by teachers and students in Victorian senior secondary mathematics in the early to mid 1990's focussed on its use as a pedagogical tool for improving student learning within existing courses, and to support student work in responding to the complexity and generality of mathematics in investigations, modelling and problem-solving tasks such as the centrally set, but school assessed, extended VCE mathematics common assessment tasks (see, for example, Tynan, 1991; Woods, 1994; Delbosc and Leigh-Lancaster, 1995). The revised VCE mathematics study, implemented from 2000, is explicit about the effective and appropriate use of technology to produce results which support learning mathematics and its application in different contexts:

The appropriate use of technology to support and develop the teaching and learning of mathematics is to be incorporated throughout each unit and course. This will include the use of some of the following technologies for various areas of study or topics: graphics calculators, spreadsheets, graphing packages, dynamic geometry systems, statistical analysis systems, and computer algebra systems. In particular, students are encouraged to use graphics calculators, spreadsheets or statistical software for probability and statistics related areas of study, and graphics calculators, dynamic geometry systems, graphing packages or computer algebra systems in the remaining areas of study systems both in the learning of new material and the application of this material in a variety of different contexts. (Board of Studies, p 12, 1999).

The Mathematical Methods (CAS) pilot study develops these considerations with respect to *congruence* between pedagogy, curriculum and assessment for computer algebra system technology. Consultation with universities and the Victorian Tertiary Admissions Centre (VTAC) took place throughout the development and accreditation of Mathematical Methods (CAS) Units 1-4 for the pilot study and in March 2001, VTAC informed the VCAA that the pilot study design had been approved by all universities for prerequisite purposes from 2003. Further details about the VCAA pilot program and its progress can be found in Leigh-Lancaster (2002) and Leigh-Lancaster (2003).

Benefits and concerns

A range of potential benefits for the use of CAS are typically articulated, including the following:

- the possibility for improved teaching of traditional mathematical topics;
- opportunities for new selection and organisation of mathematical topics;
- access to important mathematical ideas that have previously been too difficult to teach effectively;
- as a vehicle for mathematical discovery;
- extending the range of examples that can be studied;
- as a programming environment ideally suited to mathematics;
- emphasising the inter-relationships between different mathematical representations (the technology allows students to explore mathematics using different representations simultaneously);
- as an aid to preparation and checking of instructional examples;
- promoting a hierarchical approach to the development of concepts and algorithms;
- long and complex calculations can be carried out by the technology, enabling students to concentrate on the conceptual aspects of mathematics;
- the technology provides immediate feedback so that students can independently monitor and verify their ideas;

- the need to express mathematical ideas in a form understood by the technology helps students to clarify their mathematical thinking;
 - situations and problems can be modelled in more complex and realistic ways.
- (Conference proceedings, ICME 5, 1984 pp 162 – 165 and the Mathematical Association, 1997, pp 43 – 46).

For systems these potential benefits need to be considered along with the various concerns about potential negative effects that are expressed at times by academics, teachers, parents and students, including those who are nonetheless positive about the overall benefits of CAS:

- the extent to which the use of CAS may reduce students knowledge and skills with important and valued conventional by hand or mental techniques;
- how students, including those who may be less mathematically inclined, will cope with a more conceptually demanding curriculum;
- a diminished role for teachers in terms of traditional (and valued) pedagogy;
- whether appropriate cognisance has been given to the role of by hand approaches in the development of important mathematical concepts, skills and processes.

A principled and coherent response to the natural questions of *what mathematics?* (selection from discipline and domain knowledge, theory and application); for *whom?* (subsets of the cohort); *how?* (curriculum and assessment study requirements and related advice on possible pedagogies); and *why?* (rationale and purpose), is central to the responsibilities and work of curriculum and assessment authorities.

Systems and the nature of CAS use in examinations

The use of CAS in final senior secondary mathematics examinations can be precluded, permitted or assumed for part or all of these examinations (an interesting and related question for curriculum and assessment authorities is whether/how CAS might be profitably used in teaching, learning and assessment of other studies, for example, physics). Systems use a variety of approaches, with underpinning value emphases and process, knowledge and skill aims, to ensure congruence between their curriculum goals and corresponding examination structures and question design. Thus various notions such as CAS 'free', 'independent', 'advantaged', 'privileged', 'trivialised', 'neutral', 'active' (or not) and the like, can be found in the corresponding discourse, with at least as many nuances of meaning as there are notions. At this stage it appears to be the case that each system where CAS can be used, at least in part for some examinations, has a distinct structure for these examinations. These are summarised in Table 1 below:

Table 1: *comparison of systems and examination structure*

System	Examination structure
France	CAS neutral questions, unrestricted access to approved CAS for all parts of examinations, pure mathematical emphasis.
Victoria, Australia – pilot program	Assumed access to approved CAS for all parts of examinations (multiple choice, short answer and extended response), application emphasis in extended response questions. Common questions on corresponding papers for CAS and non-CAS versions of the same paper, for parallel courses with common and distinctive content.
College Board (USA) Advanced Placement	CAS 'not an advantage' questions, access to a broad range of approved graphics calculators or hand-held CAS permitted in parts, other parts

Calculus	technology free.
Denmark	Access to broad range of hand held and computer based CAS, technology assumed access parts and technology free parts, common and distinctive questions for CAS and non-CAS versions of the same paper and course.
International Baccalaureate*	Assumed access to a single approved CAS, technology assumed access parts and technology free parts.

* proposed pilot program for Higher Level course in conjunction with current review process.

Mathematical Methods (CAS) Units 3 and 4 examinations: structure and design

Mathematical Methods Units 1 – 4 and the pilot Mathematical Methods (CAS) Units 1 – 4 are parallel and like subjects with function, algebra, calculus and probability areas of study. They are considered by VTAC and the universities to be alternative but equivalent pre-requisites for the same range of university courses. The examinations for Mathematical Methods Units 3 and 4, with assumed access to a graphics calculator, and the examinations for the pilot Mathematical Methods Units (CAS) Units 3 and 4, with assumed access to CAS, are intended to make comparable demands on students. Both courses are designed to provide students with a suitable basis from which to undertake post secondary mathematics and mathematics related subjects that develop these areas of study further as well as introduce new material. One examination (Facts, skills and standard applications task) consists of multiple choice and short answer questions, while the other examination (Analysis task) consists of four extended response questions each with several parts of increasing complexity. Three of the four analysis task questions are based on an application context, and one of these questions covers material from the probability area of study.

The examinations for both subjects are time-tabled concurrently, and have the same structure and format, with substantial common material (HREF7). They include questions for which access to either CAS or a graphics calculator are not likely to be of assistance, as well as questions for which both may be of assistance, using comparable functionalities such as numerical equation solving or drawing graphs, and for which neither technology is likely to confer an advantage with respect to the other. For the 2002 examinations, together these types corresponded to about 80% of the multiple choice and extended response questions, but only around 20% of the short answer questions. These common questions provide a basis for comparison of the performance of the two cohorts.

A small number of common questions which some might consider to be 'trivialised' by access to CAS are also included. Researchers such as Herget, Heugl, Kutzler and Lehmann (2000) and Stacey (2000) have considered the issue of access to CAS 'trivialising' certain types of symbolic manipulations, and argue that these could (or should) be 'given over' to the technology, with an emphasis placed upon the use of CAS in application contexts, supported by a sense of 'algebraic expectation' or 'algebraic insight'. Other researchers, such as Drijvers and Van Herwaarden (2000) and Monaghan (2001), have discussed the complexities of such considerations, in particular, that there is a significant issue of by hand-technology interplay in the dimension of mathematical understanding and competency. Gardner (2001) in

particular cautions that cognisance of these connections is essential. Access to a given functionality or repertoire of functionalities, does not necessarily confer their effective use in practice, or understanding of important ideas and principles underpinning such use.

On the other hand, effective use of CAS can support student engagement towards completions of questions with which they might otherwise falter, either through error at an early stage or continuing reliably and accurately with the complexity of manipulations involved. Survey feedback from teachers in the pilot is supportive of this view. This assistance would have a necessary rather than sufficient impact, since CAS use in itself does not confer the insight required to apply appropriate solution processes, although it may facilitate the development of such insight through the teaching and learning process. Thus it is reasonable to anticipate that access to CAS would likely have some benefit in this regards.

Questions included in the CAS examination papers only, also cover content that has been included in the Mathematical Methods (CAS) subject, but had not previously been generally or readily available as study content for Mathematical Methods. This has typically been the case due to a combination of technical difficulty, conceptual complexity and time available to cover this material suitably. Examples of such material include continuous random variables and functional equations. Corresponding examination questions are likely to involve functions without rules being specified (such as a function f which is differentiable over a given domain); general forms involving the use of parameters, and more complex symbolic expressions, including those arising in application situations.

For other questions, the use of mental or by hands approaches will be simpler and more efficient. In many cases use of CAS will also provide students with a means of obtaining an answer, although this may not be in a convenient form, or obtainable through a simple process. A characterisation of this aspect of examination design is that students are unlikely to complete the papers in the allocated time if they rely solely on the use of either CAS or a combination of mental and by hand methods (indeed they will *not* be able to readily answer some questions, or parts of questions in the latter case). The examiners also expect students to be able to identify equivalent algebraic forms as different CAS use a variety of routines to 'simplify' expressions and carry out other computations, for example, integration of combined functions involving circular functions. The VCAA Assessment Reports for 2002 examinations provides commentary on student performance by the chief assessors for the 2002 examinations (HREF8).

Some comments and observations

The following comments and observations are based on preliminary analysis of data (see Appendix 1) related to cohort proportions of correct responses (multiple-choice questions) and cohort mean and available scores (short answer and extended response questions), discussions with panel setting chairs and chief assessors, and discussions with pilot study teachers. It should be noted that the data for Mathematical Methods is for the state-wide cohort of around 20 000 students, and that the data for the Mathematical Methods (CAS) pilot is for the 78 students from three volunteer pilot schools. These schools comprise two co-educational schools (one government sector, metropolitan school and one independent sector, regional school) and one single-sex girl's school(catholic sector, metropolitan school). Thus it is only possible to make tentative comments and observations of interest at this stage

of the pilot. For students in each VCE study, the VCAA computes a study score in the range 0 to 50, which comes from a truncated normal distribution with mean 30 and standard deviation 7. For VCE mathematics, this study score is based on two examinations, each worth 33% of the final weighting, and a school based coursework assessment score, worth 34% of the final weighting, and statistically moderated with respect to the examinations. VTAC re-scales these study scores to take into account differences in relative difficulties of studies (based on analysis of how students perform across studies) and uses these re-scaled scores in computation of a national tertiary entrance score, using a combination of best subject scores, on a scale of 0 – 100 (HREF9). For 2002, the re-scaled means and standard deviations for Mathematical Methods and Mathematical Methods (CAS) were 36.6 and 38.8 and 6.9 and 5.6 respectively. This accords with what the VCAA had anticipated, given the nature of the three volunteer schools, a slightly higher (re-scaled) mean score with a slightly smaller standard deviation. For the first two years of pilot examinations, VTAC has agreed to scale the pilot study in the same manner as Mathematical Methods.

Table 2 summarises the difference in proportion of correct responses (as a percentage of each cohort) to the 20 (out of a total of 27) common multiple choice items between Mathematical Methods (CAS) and Mathematical Methods (thus a positive difference will indicate that a higher proportion of the CAS cohort selected the correct response). The items have been classified as technology independent (I), technology of assistance but neutral with respect to graphics calculator or CAS (N) or use of CAS likely to be advantageous (C). Those items for which technology is of assistance, but that are likely to be answered efficiently by conceptual understanding, pattern recognition or mental and/or by hand approaches have been indicated by a tick (✓).

Examination 1- common questions

Table 2: *summary of differences between proportions of correct responses to common Examination 1 multiple choice items – given as number of items (question number/s)*

Item type	Negative difference*			Positive difference*		
	≥ 20%	10 to 19 %	up to 9%	up to 9 %	10 to 19 %	≥ 20%
I		1 (22)	1 (26)	3 (7, 17, 18)	3 (6, 11, 13)	
N			2 ✓ (4, 24)	4 ✓ (1, 3, 5, 25)	1 ✓ (27)	
C					1 ✓ (16)	1 ✓ (14)
					2 (9, 15)	1 (20)

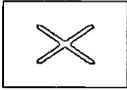
* there were no items with zero difference proportions.

The following comments are perhaps best considered as pointers to some areas of interest for further analysis within the expanded pilot program. In broad terms, the summary data in

Table 2 indicates that, on balance, this pilot cohort was not disadvantaged on these common questions when compared with the Mathematical Methods cohort. Questions 14 and 20 with high proportion of correct responses (94, 90) and large positive difference (30, 40) respectively, suggest that access to CAS can significantly improve accuracy and reliability of some symbolic computations such as simple differentiation and anti-differentiation, or at least the ability to *recognise* the correct form from a list of alternatives. On the other hand, Question 22, formulation of a definite integral to represent area between two curves, indicates that the anticipated benefits such as the view that use of CAS to carry out related computations ‘frees up’ time to place more emphasis on formulation, do not necessarily follow automatically. Similarly, Question 9 (C, 69, 55) which required students to identify the **linear** factors of $x^4 + x^3 - 3x^2 - 3x$ over R , while having a better correct response rate than the non-CAS cohort, was not as high as might have been expected if access to CAS were to have *practically* trivialised such a question. Analysis of response rates for distractors indicates that this was most likely due to students factorising over the rational field, Q , rather than the real field, R (a ‘popular’ error for both cohorts).

With respect to the common Examination 1 short answer questions there were only 4 marks of this kind (out of a total of 23 available marks). The type of question, maximum available, mean Mathematical Methods (CAS) cohort and mean Mathematical Methods cohort scores were respectively Question 5a (I, 1, 0.31, 0.24) – specifying a sequence of transformations to produce a given function rule; Question 5b (I, 2, 1.33, 1.11) – stating the domain and range of the transformed function; and question 6b (N, 1, 0.56, 0.49) – finding a numerical value for a derivative.

Examination 1 – CAS only questions

For the six CAS paper only multiple choice items, the correct response rates were Question 2: sum of solutions to a simple circular functions equation over an interval (65); Question 8: matching an explicit function rule to data (94); Question 10: functional relation (64); Question 19: identifying an unknown function in a chain rule application (86); Question 21: application of definite integrals to distance travelled for $v(t) =$  with exact value answer form (58); and Question 23: evaluating probability for continuous random variable with a linear function probability density function (60).

With respect to the CAS paper only short answer questions, it was noticeable that in Question 1a (C, ✓, 2, 1.07): finding the value of the multiplicative constant, c , so that a function with domain $[0, \infty)$ is a probability density function; and Question 1b (C, 2, 0.99): finding the median value of the random variable; while most students were able to correctly formulate a suitable definite integral expression and related equation, this did not transfer to corresponding correct calculations or evaluations to any where near the same extent. Question 3 (I, 2, 1.55): drawing the graph of $|f|$ given the graph of f (without a rule) was done well, with the two main sources of error being ‘rounding off’ of the point of non-differentiability, and incorrect concavity at the left and/or right hand ends of the curve. Question 4 required students to find the rule of a cubic polynomial function with undetermined coefficients using a combination of conditions involving the values of the function and its derivative. The three parts of the question involved formulation as a set of

simultaneous linear equations (2, 1.56), their representation in matrix form (2, 1.39) and solution (by any method) to find the rule explicitly (2, 1.13).

Examination 2 – common questions

With respect to Examination 2, 80 % of the material consisted of common extended response questions, or common parts of extended response questions. Using the same classification as previously, and consideration of the type of question, maximum available, and respective mean Mathematical Methods (CAS) cohort and mean Mathematical Methods cohort scores, the following observations can be made.

Where numerical computations and/or graph sketching is required, both cohorts performed comparably, or the CAS cohort performed slightly better, for example: Question 1a.i (N, ✓, 3, 1.61, 1.53) – drawing the graph of a transformed log function on a restricted domain in a modelling context; Question 1a. iv (I, 1, 0.21, 0.20) - stating the domain of the inverse function; Question 1a.v (N, 2, 0.71, 0.72) sketching the graph of the inverse function labelling key features.

Where algebraic manipulation was involved, access to CAS improved the reliability of correct response: Question 1a. iii (C, 2, 1.77, 1.41) – finding the rule of the inverse function; Question 1b (N, 2, 1.57, 1.15) – solving simultaneous equations to determine coefficients.

Similarly, where algebra and calculus were both involved in a theoretical context, such as in Question 3, access to CAS improved the reliability of correct response: for example, Question 3 b.i (N, ✓, 3, 2.05, 1.53) – showing a given linear function is normal to a quartic polynomial function at a specified x coordinate value. However, the earlier observation with respect to formulation involving definite integrals also receives some further support: Question 3c.i (I, 2, 0.97, 0.84) – writing down a definite integral expression for the area between the two curve and their points of intersection.

On the common parts of the extended response probability question, involving binomial, hypergeometric and normal distributions with mainly numerical calculations, (which constituted most of the available marks) the CAS cohort performed slightly better. However it was expected by examiners that there would be no appreciable difference between the two cohorts here (despite the view of some that these functions were more easily accessible on graphics calculators).

The final question, Question 4 involved a modelling context based on a transformed circular function with multiples of π in the argument, leading into a more complex product function model involving a transformed exponential function as well. While both cohorts performed similarly on some parts of this question, for example: Question 4 b (N, 1, 0.58, 0.42) – finding the first time the product function attained a particular value in its range; for each part the CAS cohort performed comparably, slightly better, or noticeably better, for example Question 4e.i (C, 2, 1.75, 1.07) – finding a symbolic expression for the product function model.

On the other hand, for a question where a simple parameter was incorporated into the model as a multiplied coefficient, and students were asked to determine the greatest (decimal) value of the parameter such that the derivative satisfied certain constraints, while the CAS cohort did perform better, neither cohort did particularly well (C, 3, 0.53, 0.35).

Examination 2 – CAS only questions

The main areas of interest here are where alternative CAS and non-CAS formulations for parts of questions have been used, or where distinctive questions have been used.

The panel setting chairs and chief assessors for Mathematical Methods (CAS) who also have considerable experience with setting and marking Mathematical Methods papers, have also commented favourably on the notable *lack* of 'blank spaces' where student responses would be expected to short answer or extended response questions, in examination scripts for the pilot CAS cohort. Question 1c was designed to require general symbolic calculation involving the use of a parameter, k , and a fixed real value T , but also to be amenable in part to conceptual graphical analysis. Question 1c.i (C, 1, 0.69) involved finding the solution to an equation, in the form $e^{fk \cdot T}$. Question 1c.i (C, \checkmark , 2, 0.21) then required students to find, for a given (but unknown) value of T , the largest value of the parameter, k , such that an equation has a solution over the domain of the underlying modelling function. The modest success rate reinforces advice from the chief examiner of the Danish pilot study to the author that, based on their experience, such questions are challenging to students, and what appears to be a small increase in complexity of question design can be a substantial increase in difficulty for students, especially where a parameter is involved. This also accords with the experience of the author as a setting panel member and assessor for Mathematical Methods Units 3 and 4 graphics calculator not permitted, graphics calculator 'neutral' examinations, and graphics calculator 'assumed access' examinations where a parameter has been used in question formulation.

In Question 3a.i (C, 1, 0.81) and Question 3 a.ii (C, 2, 1.76) which involve factorisation of a quartic polynomial function into a particular form, and finding the exact values of the zeroes, access to CAS was clearly beneficial in enabling students to obtain the required results.

Other related data

In Term 3 of 2001 (for Units 1 and 2, Year 11) and Term 3 of 2002 (for Units 3 and 4, year 12), the VCAA gathered data where students of both Mathematical Methods and Mathematical Methods (CAS) cohorts across the three pilot schools undertook an algebra and calculus skills test, related to content covered to that stage, *without* access to either graphics calculator or CAS technology. The results clearly show that students from the CAS cohort were able to perform as well or better on this material as the non-CAS cohort.

The VCAA has also collected qualitative data from teachers on their implementation of the pilot study in 2001 and 2002. VCE Mathematics units have a two-tiered assessment structure, demonstration of achievement of a set of outcomes (with specified key knowledge and key skills) for satisfactory completion of the unit, and level of performance assessment. Award of the Victoria Certificate of Education is based satisfactory completion of a minimum number of units (typically across two years) including a specified number of Unit 3 and 4 sequences (four sequences as well as a compulsory English language sequence) For example, Mathematical Methods (CAS) Units 3 and 4 form a sequence. At the Unit 3 and 4 level, students will not receive a study score for a sequence unless they have satisfactorily completed *both* of its units. Coursework assessment is also based on the outcomes, so their associated key knowledge and key skills are also assessed through the coursework assessment tasks for Units 3 and 4.

The outcomes for Mathematical Methods (CAS) Units 1 – 4 clearly specify expected mental, by hand and CAS concepts, skills and processes, and teachers, parents and students from pilot schools have clearly affirmed their valuing for the effective development of important

mental and by hand algebra and calculus skills. Thus, in discussion of the role of teaching by hand algebraic manipulations and differentiation rules for combined functions, pilot teachers felt that this was still important (yet were comfortable about students choosing when to use CAS or by hand approaches when tackling problems), for the following sorts of reasons, as summarised by one of the original pilot teachers who is a highly regarded and expert teacher with considerable experience over many years in the use of technology in school mathematics:

- so that students can see a process first (or else they think it is magic);
- so students can recognise equivalent forms of a solution;
- so that students can access a range of methods (for versatility);
- so that students can make an informed choice about the tool they use (head/hand/CAS or combination thereof);
- so that students can develop a sense of whether a result is plausible.

Teachers and students from the pilot study also reported affirmation of a range of the potential benefits outlined earlier, in particular a more in depth treatment of existing material, access to new and interesting content, enhanced engagement and persistence of students in mathematical work, increased accuracy and reliability of student work, including in application contexts, and improved understanding of the concepts of function and variable, and facility with symbolic notation and exact values. A greater confidence and independence of student work in mathematical activity has also been noted. Certainly, teachers have commented that their own mathematical horizons have been extended, and that they have developed an expanded pedagogical repertoire (see Garner, 2003) as a consequence of their involvement in the pilot study.

Biographical details

David is the Mathematics Manager at the Victorian Curriculum and Assessment Authority, and the project manager of the VCAA Mathematical Methods (CAS) pilot study 2001 – 2005. He has taught secondary mathematics for around 20 years, including 12 years as a head of faculty. During this time he has been extensively involved in curriculum and teacher professional development, examinations and school based tasks. He has a long-standing interest in the nature of mathematical inquiry, related teaching and learning approaches and assessment tasks.

David's mathematical background is in pure mathematics, in particular mathematical logic and the history and philosophy of mathematics. He has worked with the application of technology in mathematics throughout his career, and has used the CAS *Mathematica* with students from Years 9 – 12 at Kingswood College, Victoria from 1993 – 1998.

References

- Board of Studies. (1999). *VCE Study design – Mathematics*. Carlton: Author.
- Del Bosc, M. & Leigh-Lancaster, D. (1995). Specialist Mathematica. In *Celebrating Mathematics Learning*. Mathematical Association of Victoria. Melbourne.
- Drijvers, P. & Van Herwaarden, O. (2000). Instrumentation of ICT-tools: the case of algebra in a computer algebra environment. *International Journal of Computer algebra in Mathematics Education* 7(4), pp 255 - 275.
- Gardner, T. (2001). Education or CAstration?. *Micromath*, Spring 17 (1), pp 6 – 8.

- Garner, S. (2003). How has the use of the CAS calculator affected the teaching and learning of Maths in the Year 12 classroom? In *Mathematics~Making Waves*. Ed. M. Goos. Australian Association of Mathematics Teachers. Adelaide.
- Herget, W., Heugl, H., Kutzler, B. & Lehmann, E. (2000). Indispensable manual calculation skills in a CAS environment. *Micro math, autumn 16* (3) pp 8 -17.
- ICME 5. (1984). *Proceedings of the Fifth International Congress of Mathematics Education (ICME 5)*. Ed M. Carss. Birkhauser. Boston.
- Leigh-Lancaster, D. (2002). The Victorian Curriculum and Assessment Authority (VCAA) Mathematical Methods (CAS) pilot study 2001 – 2005. *Australian Senior Mathematics Journal*, Volume 16, Number 2, pp 20 – 28.
- Leigh-Lancaster, D. (2003). The Victorian Curriculum and Assessment Authority (VCAA) Mathematical Methods (CAS) pilot study 2001 – 2005. In *Mathematics Making Waves*, Ed. M Goos. Proceedings of the 19th Biennial Conference. Australian Association of Mathematics Teachers, Adelaide.
- Leigh-Lancaster, D. & Stephens, M. (1997). Anticipating the impact of widely available access to Computer Algebra Systems on Teaching and Assessment. In *Mathematics: the next millennium*. Ed. K. Baldwin and J. Roberts. Australian Association of Mathematics Teachers, Adelaide.
- Leigh-Lancaster, D. & Stephens, M. (2001). Responding to the impact of increasing accessibility of Computer Algebra Systems (CAS) on national and other school systems at the senior secondary level. In Proceedings of the 12th ICMI Study Conference: *The Future of the Teaching and Learning of Algebra*. Ed H. Chick. Melbourne. The University of Melbourne.
- Monaghan, J. (2001). Reaction to indispensable manual calculation skills in a CAS environment. *Micromath, Spring 17* (1), pp 9 – 11.
- Stacey, K., McCrae, B., Chick, H., Asp, G., & Leigh-Lancaster, D. (2000). Research-led policy change for technologically-active senior mathematics assessment. In J. Bana and A. Chapman (Eds.), *Mathematics Education Beyond 2000* (Proceedings of the 23rd annual conference of the Mathematics Education Research Group of Australasia, pp. 572-579). Freemantle. MERGA.
- The Mathematical Association. (1996). *Symbolic Manipulation by Computers and calculators*. Ed. A. Oldknow and J. Flower. The Mathematical Association. London.
- Tynan, D. (1991). Symbolic Algebra Processors - A Manipulative Revolution. In *Mathematics: Ideas*. Eds. J. O'Reilly & S. Wettenhall. Melbourne. Mathematical Association of Victoria.
- Woods, N. (1994). *Mathematics software and the effect on curriculum development*. In *Mathematics Without Limits*. Eds. C. Beesey & D. Rasmussen. Melbourne. Mathematical Association of Victoria.

HREF1:

Paper presented at the third CAME Conference, 23 – 24 June, Rheims, France.

http://www.math.ohio-state.edu/~waitsb/papers/t3_posticme2000/ - Teachers Teaching with Technology, Post ICME 2000 Conference, Japan, Mathematics Department, Ohio State University.

HREF2:

<http://www.mmv.vic.gov.au> – Reality Bytes, State Government of Victoria.

HREF3:

<http://www.otte.vic.gov.au/publications/KnowledgeandSkills/> - Ministerial Statement on Knowledge and Skills for the Innovation Society, State Government of Victoria.

HREF4:

<http://www.vcaa.vic.edu.au/board/pdfs/stratpla.pdf> - *Strategic Plan 2002 – 4*, VCAA.

HREF5:

<http://www.vcaa.vic.edu.au/vce/studies/MATHS/caspilot.htm> -the Mathematical Methods (CAS) pilot study website, VCAA.

HREF6:

<http://www.edfac.unimelb.edu.au/DSME/CAS-CAT> - the CAS - CAT project website, University of Melbourne.

HREF7:

<http://www.vcaa.vic.edu.au/VCE/Exams/subject.htm#maths02> – mathematics examinations, VCAA.

HREF8:

<http://www.vcaa.vic.edu.au/VCE/Assessment/AssessReports/subject.htm#maths> – mathematics assessment reports, VCAA.

HREF9:

http://www.vtac.edu.au/articles/scaling_report.pdf Victorian Tertiary Admissions Centre.

Each of these sites was accessed at 28/03/03.

BEST COPY AVAILABLE

Mathematical Methods and Mathematical Methods (CAS) examinations 2002 –common questions

Table 1 summarises the totals of marks from common questions across each component of the examinations, and the aggregate total across all components.

Table 1

Component	Nature	Proportion of marks
Examination 1 – Part I	Multiple choice	21 out of 27 ($\approx 78\%$)
Examination 1 – Part II	Short answer	4 out of 23 ($\approx 17\%$)
Examination 2	Extended response	44 out of 55 (80 %)
Aggregate		69 out of 105 ($\approx 66\%$)

Table 2 lists the actual common questions for each part, and associated marks.

Component	Questions	Marks
Examination 1 – Part I	1, 3, 4, 5, 6, 7, 9, 11, 12, 13, 14, 15, 16, 17, 18, 20, 22, 24, 25, 26, 27	1 mark each (Total 21 marks)
Examination 1 – Part II	Question 5 parts a and b Question 6 part b	1 + 2 marks 1 mark (Total 4 marks)
Examination 2	Question 1 parts ai, aii, aiii, aiv and av Question 1 part b Question 2 MM part a = MM(CAS) part b Question 2 MM part b = MM(CAS) part c Question 2 MM part d = MM(CAS) part d Question 3 parts bi, bii Question 3 parts ci, cii Question 4 part ai Question 4 MM part bi = MM(CAS) part b Question 4 MM part bii = MM(CAS) part c Question 4 MM part biii = MM(CAS) part d Question 4 MM part ci = MM(CAS) part ei Question 4 MM part cii = MM(CAS) part eii	3 + 1 + 2 + 1 + 2 marks 2 marks 2 marks 4 marks 4 marks 3 + 4 marks 2 + 1 marks 1 mark 1 mark 1 mark 2 marks 2 marks 3 marks

	Question 4 MM part e = MM(CAS) part f	3 marks
		(Total 44 marks)

Mathematical Methods and Mathematical Methods (CAS) examinations 2002 – cohort comparative performance on common questions

The following classifications for type of item have been used:

- I** Technology independent - for example a conceptual question or analysis of a non-scaled graph without rules for functions
- N** Technology of assistance, but neutral with respect to graphics calculator or CAS calculator use - for example, sketch of a graph or numerical solution to an equation or numerical evaluation of a derivative or definite integral
- C** Use of a CAS calculator likely to be an advantage – for example, symbolic manipulation for finding exact value roots of an equations, factors of an expression, symbolic expressions for a derivative or anti-derivative.

19

Questions where technology is of assistance, but that are likely to be answered efficiently by conceptual understanding, pattern recognition, straightforward mental or by hand approaches have been indicated by a tick (✓) in the type column following.

Examination 1 Part I: Multiple choice items (78% common material)

Question	Description	Type	% correct responses	% correct responses	% correct responses

			MM (CAS) cohort	MM cohort	MM (CAS) - MM
1	Identify rule of circular function that matches a given graph.	N	88	86	+2
3	Given rule of a transformed circular function in a modelling context, identify maximum value on a specified domain.	N	79	73	+5
4	Given graph of a quartic polynomial function identify possible rule for the function.	N	68	73	-5

Question	Description	Type	% correct responses MM (CAS) cohort	% correct responses MM cohort	% correct responses MM (CAS) - MM
5	Given graph of a transformed basic hyperbola, find the values of the parameters that define the corresponding function rule.	N ✓	92	84	+8
6	Given the graph of a function with unspecified rule $f(x)$, identify the corresponding graph for $f(x)$.	I	67	55	+12
7	Given the graph of undetermined 1 – 1 function on 1:1 axes scales, identify the graph of the inverse function.	I	86	82	+4

15	Find the normal to a curve whose rule is a simple product function at a given exact value point.	Cor N	59	47	+12
16	Find the exact rate of change for a simple transformed exponential function base e at a given exact point.	Cor N	82	72	+10
17	Identify substitutions necessary for correct application of linear approximation formula.	I	67	61	+6
18	For an unspecified function f , given a combinations of information about the function at points and the derivative at points and over intervals, find the corresponding graph for f .	I	62	58	+4

Examination 2 – extended response questions with several parts of increasing complexity (80% common material)

Question 1 is a modelling context based around a transformed natural logarithm function with rule of the form $f(x) = a - b \log_e(x)$ on a restricted domain.

Question part	Description	Type	Maximum available marks	Mean score MM (CAS) cohort	Mean score MM cohort
1a.i	Sketch the graph of a transformed log function (decimal value coefficients) on a restricted domain in a modelling context, labelling any asymptotes and any endpoints with their coordinates.	N ✓	3	1.61	1.53
1a.ii	Explain why the function has an inverse function.	I	1	0.81	0.63

1a.iii	Find the rule of the inverse function.	C		2	1.77	1.41
1a.iv	State the domain of the inverse function.	I		1	0.21	0.20
1a.v	Sketch the graph of the inverse function, labelling any asymptotes and any endpoints with their coordinates.	N		2	0.71	0.72
1b	Solve simultaneous equations to determine new coefficient values for different modelling data.	N	✓	2	1.57	1.15

Question 3 is a theoretical context involving a quartic polynomial function, differentiation and integration, tangents and normals, points of intersection and area between two curves.

Question part	Description	Type	Maximum available marks	Mean score MM (CAS) cohort	Mean score MM cohort
3 b.i	Show that a given linear function is normal to the quartic polynomial at a specified x coordinate.	N ✓	3	2.05	1.53
3 b.ii	Show that this normal is tangent to the quartic polynomial at another point, and determine the exact value coordinates of that point.	N ✓	4	2.19	1.46
3 c.i	Write down a definite integral expression for the area between the quartic and the linear (normal/tangent) functions defined by their points of intersection.	I	2	0.97	0.84

3 c. ii	Evaluate the definite integral to a specified accuracy.	N		1	0.41	0.26
---------	---	---	--	---	------	------

Question 4 is a modelling context involving firstly a height-time transformed sin function with multiple of π in the argument of the function, and secondly a product function involving an exponential function and a transformed circular function. The problem involves solving equations, rates of change and differentiation.

Question part	Description	Type	Maximum available marks	Mean score MM (CAS) cohort	Mean score MM cohort
4 a.i	Find the maximum value of a transformed sin function in a modelling context.	N ✓	1	0.95	0.84
4 b (MM 4 b.i)	Find the first time a particular height is reached for the product exponential and transformed sin function.	N	1	0.58	0.42
4c (MM 4 b.ii)	Find how many times a particular height is achieved during a specified time interval, for the product exponential and transformed sin function.	N ✓	1	0.73	0.60

4d (MM 4 b.iii)	Given a specific height value find the time elapsed from start until that height is reached, for the product exponential and transformed sin function.	N	2	1.60	1.06
4 e.i (MM 4 c.i)	Find a symbolic expression for dy/dt , the product exponential and transformed sin function.	C	2	1.75	1.07
4 e.ii (MM 4 c.ii)	Use dy/dt to write down an equation, one solution of which gives the value of t for the minimum height, and the value of the minimum height (decimal value answers required).	C	3	1.66	0.95

Plenary paper at 'CARE, Reims, France
23-24 June 2003



U.S. Department of Education
Office of Educational Research and Improvement (OERI)
National Library of Education (NLE)
Educational Resources Information Center (ERIC)

Selec 0319
ERIC

REPRODUCTION RELEASE
(Specific Document)

I. DOCUMENT IDENTIFICATION:

Title: <i>THE VICTORIAN CURRICULUM AND ASSESSMENT AUTHORITY MATHEMATICAL METHODS COMPUTER ALGEBRA PILOT STUDY AND EXAMINATIONS</i>	
Author(s): <i>DAVID LEIGH-LANCASTER</i>	
Corporate Source: <i>VICTORIAN CURRICULUM AND ASSESSMENT AUTHORITY *</i>	Publication Date: <i>23/06/03</i>

II. REPRODUCTION RELEASE:

In order to disseminate as widely as possible timely and significant materials of interest to the educational community, documents announced in the monthly abstract journal of the ERIC system, *Resources in Education (RIE)*, are usually made available to users in microfiche, reproduced paper copy, and electronic media, and sold through the ERIC Document Reproduction Service (EDRS). Credit is given to the source of each document, and, if reproduction release is granted, one of the following notices is affixed to the document.

If permission is granted to reproduce and disseminate the identified document, please CHECK ONE of the following three options and sign at the bottom of the page.

The sample sticker shown below will be affixed to all Level 1 documents

The sample sticker shown below will be affixed to all Level 2A documents

The sample sticker shown below will be affixed to all Level 2B documents

PERMISSION TO REPRODUCE AND DISSEMINATE THIS MATERIAL HAS BEEN GRANTED BY

Sample

TO THE EDUCATIONAL RESOURCES INFORMATION CENTER (ERIC)

1

PERMISSION TO REPRODUCE AND DISSEMINATE THIS MATERIAL IN MICROFICHE AND IN ELECTRONIC MEDIA FOR ERIC COLLECTION SUBSCRIBERS ONLY HAS BEEN GRANTED BY

Sample

TO THE EDUCATIONAL RESOURCES INFORMATION CENTER (ERIC)

2A

PERMISSION TO REPRODUCE AND DISSEMINATE THIS MATERIAL IN MICROFICHE ONLY HAS BEEN GRANTED BY

Sample

TO THE EDUCATIONAL RESOURCES INFORMATION CENTER (ERIC)

2B

Level 1

Level 2A

Level 2B

Check here for Level 1 release, permitting reproduction and dissemination in microfiche or other ERIC archival media (e.g., electronic) and paper copy.

Check here for Level 2A release, permitting reproduction and dissemination in microfiche and in electronic media for ERIC archival collection subscribers only.

Check here for Level 2B release, permitting reproduction and dissemination in microfiche only.

Documents will be processed as indicated provided reproduction quality permits. If permission to reproduce is granted, but no box is checked, documents will be processed at Level 1.

I hereby grant to the Educational Resources Information Center (ERIC) nonexclusive permission to reproduce and disseminate this document as indicated above. Reproduction from the ERIC microfiche or electronic media by persons other than ERIC employees and its system contractors requires permission from the copyright holder. Exception is made for non-profit reproduction by libraries and other service agencies to satisfy information needs of educators in response to discrete inquiries.

Sign here → please

Signature: <i>D. Leigh-Lancaster</i>	Printed Name/Position/Title: <i>MATHEMATICS MANAGER</i>	
Organization/Address: <i>VCAA, 41 ST ANDREWS PLACE</i>	Telephone: <i>+61 3 9651 4537</i>	FAX: <i>+61 3 9651 4324</i>
<i>EAST MERRBOURNE VICTORIA</i>	E-Mail Address:	Date: <i>11/08/03</i>
<i>3002 AUSTRALIA</i>		

(over)

leigh-lancaster.david.d@edumail.vic.gov.au

Share Your Ideas With Colleagues Around the World

Submit your conference papers or other documents to the world's largest education-related database, and let ERIC work for you.

The Educational Resources Information Center (ERIC) is an international resource funded by the U.S. Department of Education. The ERIC database contains over 850,000 records of conference papers, journal articles, books, reports, and non-print materials of interest to educators at all levels. Your manuscripts can be among those indexed and described in the database.

Why submit materials to ERIC?

- **Visibility.** Items included in the ERIC database are announced to educators around the world through over 2,000 organizations receiving the abstract journal, *Resources in Education (RIE)*; through access to ERIC on CD-ROM at most academic libraries and many local libraries; and through online searches of the database via the Internet or through commercial vendors.
- **Dissemination.** If a reproduction release is provided to the ERIC system, documents included in the database are reproduced on microfiche and distributed to over 900 information centers worldwide. This allows users to preview materials on microfiche readers before purchasing paper copies or originals.
- **Retrievability.** This is probably the most important service ERIC can provide to authors in education. The bibliographic descriptions developed by the ERIC system are retrievable by electronic searching of the database. Thousands of users worldwide regularly search the ERIC database to find materials specifically suitable to a particular research agenda, topic, grade level, curriculum, or educational setting. Users who find materials by searching the ERIC database have particular needs and will likely consider obtaining and using items described in the output obtained from a structured search of the database.
- **Always "In Print."** ERIC maintains a master microfiche from which copies can be made on an "on-demand" basis. This means that documents archived by the ERIC system are constantly available and never go "out of print." Persons requesting material from the original source can always be referred to ERIC, relieving the original producer of an ongoing distribution burden when the stocks of printed copies are exhausted.

So, how do I submit materials?

- Complete and submit the *Reproduction Release* form printed on the reverse side of this page. You have two options when completing this form: If you wish to allow ERIC to make microfiche and paper copies of print materials, check the box on the left side of the page and provide the signature and contact information requested. If you want ERIC to provide only microfiche or digitized copies of print materials, check the box on the right side of the page and provide the requested signature and contact information. If you are submitting non-print items or wish ERIC to only describe and announce your materials, without providing reproductions of any type, please contact ERIC/CSMEB as indicated below and request the complete reproduction release form.
- Submit the completed release form along with two copies of the conference paper or other document being submitted. There must be a separate release form for each item submitted. Mail all materials to the attention of Niqui Beckrum at the address indicated.

For further information, contact...

Niqui Beckrum
Database Coordinator
ERIC/CSMEB
1929 Kenny Road
Columbus, OH 43210-1030

1-800-276-0462
(614) 292-6717
(614) 292-0263 (Fax)
ericse@osu.edu (e-mail)

VICTORIAN CURRICULUM AND ASSESSMENT AUTHORITY



41 St Andrews Place
East Melbourne
Victoria 3002 Australia

David Leigh-Lancaster
Key Learning Area Manager

Mathematics

TELEPHONE +61 3 9651 4537

FACSIMILE +61 3 9651 4324

EMAIL leigh-lancaster.david.d@edumail.vic.gov.au

